

# Understanding the $D_{sJ}^+(2317)$ and $D_{sJ}^+(2460)$ with Sum Rules in HQET

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## Abstract

In the framework of heavy quark effective theory we use QCD sum rules to calculate the masses of the  $\bar{c}s$  ( $0^+, 1^+$ ) and ( $1^+, 2^+$ ) excited states. The results are consistent with that the states  $D_{sJ}(2317)$  and  $D_{sJ}(2460)$  observed by BABAR and CLEO are the  $0^+$  and  $1^+$  states in the  $j_l = \frac{1}{2}^+$  doublet.

PACS number: 12.39.Hg, 13.25.Hw, 13.25.Ft, 12.38.Lg

## 1 Introduction

Recently BaBar Collaboration announced a positive-parity narrow state with a rather low mass  $2317 \pm 3$  MeV in the  $D_s^+(1969)\pi$  channel [1], which was confirmed by CLEO [2] and BELLE [3] later. Because of its low mass and decay angular distribution, its  $J^P$  is believed to be  $0^+$ . In the same experiment CLEO [2] observed a state at 2460 MeV with the possible spin-parity  $J^P = 1^+$  in the  $D_s^*\pi$  channel. BaBar [1] also found a signal near there. Since these two states lie below  $DK$  and  $D^*K$  threshold respectively, the potentially dominant s-wave decay modes  $D_{sJ}(2317) \rightarrow D_s K$  etc are kinematically forbidden. Thus the radiative decays and isospin-violating strong decays become favorable decay modes. The later decay goes in two steps with the help of virtual  $D_s\eta$  intermediate states,  $D_{sJ}(2317) \rightarrow D_s\eta \rightarrow D_s\pi^0$  where the second step arises from the tiny  $\eta$ - $\pi^0$  isospin-violating mixing due to  $m_u \neq m_d$ .

The experimental discovery of these two states has triggered heated debate on their nature recently in literature. The key point is to understand their low masses. The  $D_{sJ}(2317)$  mass is significantly lower than the values of  $0^+$  mass in the range of 2.4 – 2.6 GeV calculated in quark models of [4]. The model using the heavy quark mass expansion of the relativistic Bethe-Salpeter equation in [5] predicted a lower value 2.369 GeV of  $0^+$  mass which is still 50 MeV higher than the experimental data. Bardeen, Eichten and Hill interpreted them as the  $\bar{c}s$  ( $0^+, 1^+$ ) spin doublet as the parity conjugate states of the

$(0^-, 1^-)$  doublet in the framework of chiral symmetry[6]<sup>1</sup> (See also Ref. [7]). A quark-antiquark picture was also advocated by Colangelo and Fazio [9], Cahn and Jackson [10], Godfrey [11]. Based on such a "conventional" picture, the various decay modes were discussed in Refs. [6, 9, 11].

Apart from the quark-antiquark interpretation,  $D_{sJ}(2317)$  was suggested to be a four-quark state by Cheng and Hou [12] and Barnes, Close and Lipkin [13]. Szczepaniak even argued that it could be a strong  $D\pi$  atom [14]! But we think it would be very exceptional for a molecule or atom to have a binding energy as large as 40 MeV.

Van Beveren and Rupp [15] argued from the experience with  $a_0/f_0(980)$  that the low mass of  $D_{sJ}(2317)$  could arise from the mixing between  $DK$  continuum and the lowest scalar nonet. In this way the  $0^+ \bar{c}s$  state is artificially pushed much lower than that expected from quark models.

The recent lattice calculation suggests a value around 2.57 GeV for  $0^+$  state mass [16], much larger than the experimentally observed  $D_{sJ}(2317)$  and compatible with quark model predictions. The conclusion in Ref. [16] is that  $D_{sJ}(2317)$  might receive a large component of  $DK$  and the physics might resemble  $a_0/f_0(980)$ . Such a large  $DK$  component makes lattice simulation very difficult.

In this paper we shall use QCD sum rules [20] in the framework of the heavy quark effective theory (HQET) [21] to extract the masses since HQET provides a systematic method to compute the properties of heavy hadrons containing a single heavy quark via the  $1/m_Q$  expansion, where  $m_Q$  is the heavy quark mass. The masses of ground state heavy mesons have been studied with QCD sum rules in HQET in [22, 23, 24]. In [17, 18, 19] masses of the lowest excited non-strange heavy meson doublets  $(0^+, 1^+)$  and  $(1^+, 2^+)$  were studied with the sum rules in HQET up to the order of  $\mathcal{O}(1/m_Q)$ . These masses were also analyzed in the earlier works [25] with sum rules in full QCD. In this short note we extend the same formalism in [17, 18] to include the light quark mass in calculating the  $\bar{c}s$  mesons. We shall also use more stringent criteria for the stability windows of the sum rules in the numerical analysis.

## 2 $m_M - m_Q$ at the leading order of HQET

The proper interpolating current  $J_{j,P,j_\ell}^{\alpha_1 \dots \alpha_j}$  for a state with the quantum numbers  $j, P, j_\ell$  in HQET was given in [17]. These currents were proved to satisfy the following conditions

$$\langle 0 | J_{j,P,j_\ell}^{\alpha_1 \dots \alpha_j}(0) | j', P', j'_\ell \rangle = f_{Pj_\ell} \delta_{jj'} \delta_{PP'} \delta_{j_\ell j'_\ell} \eta^{\alpha_1 \dots \alpha_j}, \quad (1)$$

$$i \langle 0 | T \left( J_{j,P,j_\ell}^{\alpha_1 \dots \alpha_j}(x) J_{j',P',j'_\ell}^{\dagger \beta_1 \dots \beta_{j'}}(0) \right) | 0 \rangle = \delta_{jj'} \delta_{PP'} \delta_{j_\ell j'_\ell} (-1)^j \mathcal{S} g_t^{\alpha_1 \beta_1} \dots g_t^{\alpha_j \beta_j} \\ \times \int dt \delta(x - vt) \Pi_{P,j_\ell}(x) \quad (2)$$

in the  $m_Q \rightarrow \infty$  limit, where  $\eta^{\alpha_1 \dots \alpha_j}$  is the polarization tensor for the spin  $j$  state,  $v$  is the velocity of the heavy quark,  $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$  is the transverse metric tensor,  $\mathcal{S}$

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<sup>1</sup>The existence of parity doublets has also been shown by combining chiral symmetry and heavy quark symmetry in Bethe-Salpeter approach in [8].

denotes symmetrizing the indices and subtracting the trace terms separately in the sets  $(\alpha_1 \cdots \alpha_j)$  and  $(\beta_1 \cdots \beta_j)$ ,  $f_{P,j_\ell}$  and  $\Pi_{P,j_\ell}$  are a constant and a function of  $x$  respectively which depend only on  $P$  and  $j_\ell$ .

We consider the correlator

$$\Pi(\omega) = i \int d^4x e^{ikx} \langle 0 | T \left( J_{j,P,j_\ell}^{\alpha_1 \cdots \alpha_j}(x) J_{j',P',j'_\ell}^{\dagger \beta_1 \cdots \beta_{j'}}(0) \right) | 0 \rangle \quad (3)$$

where  $\omega = 2v \cdot k$ . It can be written as

$$\Pi(\omega) = \frac{f_{P,j_\ell}^2}{2\bar{\Lambda}_{j,P,j_\ell} - \omega} + \text{higher states} , \quad (4)$$

Where  $\bar{\Lambda}_{j,P,j_\ell} = \lim_{m_Q \rightarrow \infty} (m_{M_{j,P,j_\ell}} - m_Q)$ . On the other hand, it will be calculated in terms of quarks and gluons. Invoking Borel transformation to Eq. (3) we get

$$f_{P,j_\ell}^2 e^{-\frac{2\bar{\Lambda}_{j,P,j_\ell}}{T}} = \frac{1}{\pi} \int_{2m_q}^{\omega_c} \rho(\omega) e^{-\frac{\omega}{T}} + \text{condensates} . \quad (5)$$

where  $m_q$  is the light quark mass,  $\rho(\omega)$  is the perturbative spectral density and  $\omega_c$  is the threshold parameter used to subtract the higher state contribution with the help of quark-hadron duality assumption.

We shall study the low-lying  $(0^+, 1^+)$ ,  $(1^+, 2^+)$  ( $\bar{c}s$ ) states. The values of various QCD condensates are

$$\begin{aligned} \langle \bar{s}s \rangle &= -(0.8 \pm 0.1) * (0.24 \text{ GeV})^3 , \\ \langle \alpha_s GG \rangle &= 0.038 \text{ GeV}^4 , \\ m_0^2 &= 0.8 \text{ GeV}^2 . \end{aligned} \quad (6)$$

We use  $m_s(1\text{GeV}) = 0.15 \text{ GeV}$  for the strange quark mass in the  $\bar{M}S$  scheme. We use  $\Lambda_{QCD} = 375 \text{ MeV}$  for three active flavors and  $\Lambda_{QCD} = 220 \text{ MeV}$  for four active flavors. The sum rules with massless light quarks have been obtained in [18, 17].

In the numerical analysis of the QCD sum rules we require that the high-order power corrections be less than 30% of the perturbation term. This condition yields the minimum value  $T_{min}$  of the allowed Borel parameter. We also require that the pole term, which is equal to the sum of the cut-off perturbative term and the condensation terms, is larger than 60% of the perturbative term, which leads to the maximum value  $T_{max}$  of the allowed  $T$ . Thus we have the working interval  $T_{min} < T < T_{max}$  for a fixed  $\omega_c$ . If  $T_{min} \geq T_{max}$ , we are unable to extract useful information from such a sum rule. In the ideal case, the difference between the meson mass  $m_M$  and the heavy quark mass  $m_Q$  (or other observables) is almost independent on  $T$  for certain values of  $\omega_c$ . Namely, the dependence on both the Borel parameter and continuum threshold is minimum. In realistic cases, the variation of a sum rule with both  $T$  and  $\omega_c$  will contribute to the errors of the extracted value, together with the truncation of the operator product expansion and the uncertainty of vacuum condensate values.

In subsections 2.1 and 2.2 we give sum rules for  $(0^+, 1^+)$  and  $(1^+, 2^+)$  doublets using interpolating currents with derivatives, respectively. For comparison we discuss sum rules for  $(0^+, 1^+)$  using interpolating currents without derivatives in subsection 2.3.

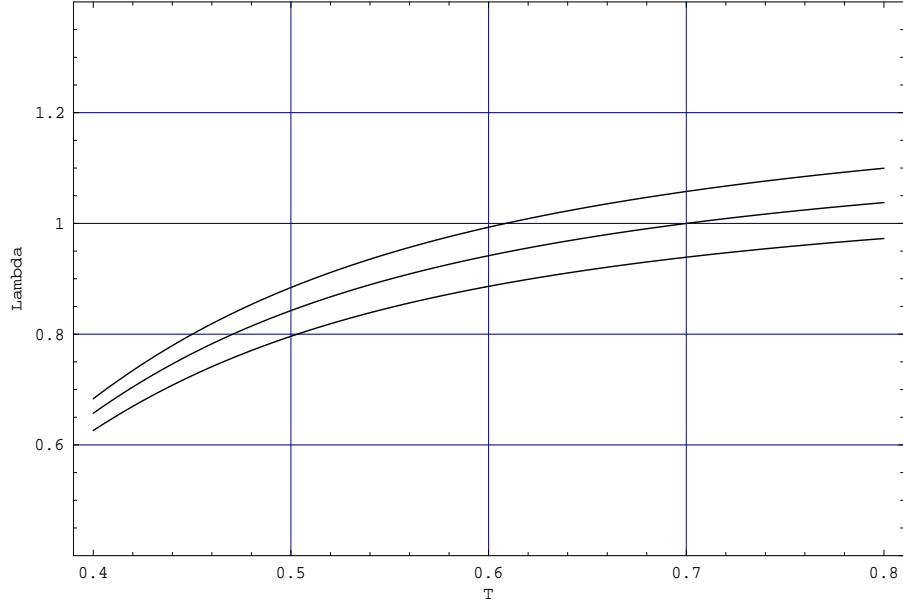


Figure 1: The variation of  $\bar{\Lambda}(\frac{1}{2}^+)$  (in unit of GeV) of the  $(0^+, 1^+)$  doublet with  $T$  and  $\omega_c$  for the derivative currents. The vertical and horizontal axes correspond to  $\bar{\Lambda}$  and  $T$ . From top to bottom, the curves correspond to  $\omega_c$  being 3.1, 2.9, 2.7 GeV respectively.

## 2.1 $(0^+, 1^+)$ Doublet With Derivative Currents

We employ the following interpolating currents [17]

$$\begin{aligned} J_{0,+2}'^\dagger &= \frac{1}{\sqrt{2}} \bar{h}_v(-i) \mathcal{P}_{ts}, \\ J_{1,+2}'^{\alpha\dagger} &= \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma_t^\alpha(-i) \mathcal{P}_{ts}. \end{aligned} \quad (7)$$

where  $D_t^\mu = D^\mu - (v \cdot D) v^\mu$ , with  $D^\mu = \partial^\mu - ig A^\mu$  being the gauge-covariant derivative. We obtain the sum rule relevant to  $\bar{\Lambda}$

$$\begin{aligned} f_{+,1/2}'^2 e^{-2\bar{\Lambda}/T} &= \frac{3}{64\pi^2} \int_{2m_s}^{\omega_c} [\omega^4 + 2m_s \omega^3 - 6m_s^2 \omega^2 - 12m_s^3 \omega] e^{-\omega/T} d\omega \\ &\quad - \frac{1}{16} m_0^2 \langle \bar{s}s \rangle + \frac{3}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{16\pi} \langle \alpha_s G^2 \rangle. \end{aligned} \quad (8)$$

Taking the derivative of the logarithm of the above equation with respect to  $(\frac{1}{T})$  one obtains the sum rule for  $\bar{\Lambda}$ . Substituting the obtained value of  $\bar{\Lambda}$  in (8) one obtains the sum rule for  $f_{+,1/2}'$ . In Figure 1, the variation of  $\bar{\Lambda}(1/2^+)$  with  $T$  and  $\omega_c$  is shown. According to the criteria stated above the working range is  $0.38 < T < 0.58$  GeV. We have

$$\begin{aligned} \bar{\Lambda}(1/2^+) &= (0.86 \pm 0.10) \text{ GeV}, \\ f_{+,1/2}' &= (0.31 \pm 0.05) \text{ GeV}^{5/2}, \end{aligned} \quad (9)$$

where the central value corresponds to  $T = 0.52$  GeV and  $\omega_c = 2.9$  GeV.

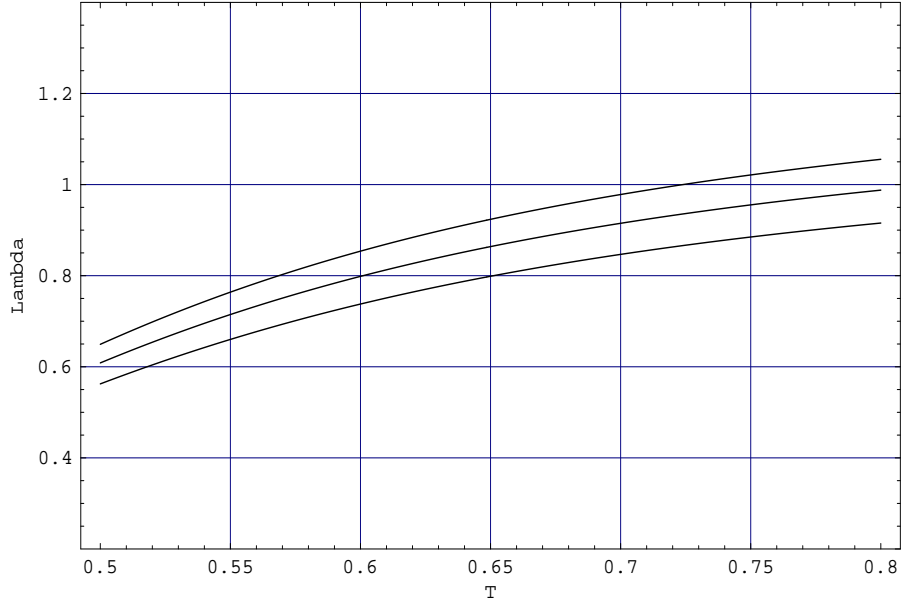


Figure 2: The variation of  $\bar{\Lambda}(\frac{3}{2}^+)$  (in unit of GeV) of the  $(1^+, 2^+)$  doublet with  $T$  and  $\omega_c$ . From top to bottom,  $\omega_c = 3.2, 3.0, 2.8$  GeV respectively.

## 2.2 $(1^+, 2^+)$ Doublet

For the  $(1^+, 2^+)$  doublet, by using the following interpolating currents [17]

$$\begin{aligned} J_{1,+1}^{\dagger\alpha} &= \sqrt{\frac{3}{4}} \bar{h}_v \gamma^5 (-i) \left( \mathcal{D}_t^\alpha - \frac{1}{3} \gamma_t^\alpha \not{p}_t \right) q, \\ J_{2,+1}^{\dagger\alpha_1\alpha_2} &= \sqrt{\frac{1}{2}} \bar{h}_v \frac{(-i)}{2} \left( \gamma_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} + \gamma_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} - \frac{2}{3} g_t^{\alpha_1\alpha_2} \not{p}_t \right) q, \end{aligned} \quad (10)$$

we obtain the sum rule

$$\begin{aligned} f_{+,3/2}^2 e^{-2\bar{\Lambda}/T} &= \frac{1}{64\pi^2} \int_{2m_s}^{\omega_c} [\omega^4 + 2m_s\omega_c^3 - 6m_s^2\omega^2 - 12m_s^3\omega] e^{-\omega/T} d\omega \\ &\quad - \frac{1}{12} m_0^2 \langle \bar{s}s \rangle - \frac{1}{32} \langle \frac{\alpha_s}{\pi} G^2 \rangle T + \frac{1}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{48\pi} \langle \frac{\alpha_s}{\pi} G^2 \rangle. \end{aligned} \quad (11)$$

From the sum rule, the variation of  $\bar{\Lambda}(3/2^+)$  with  $T$  and  $\omega_c$  is plotted in Figure 2. The working range is  $0.55 < T < 0.65$  GeV. We have

$$\begin{aligned} \bar{\Lambda}(3/2^+) &= (0.83 \pm 0.10) \text{ GeV}, \\ f_{+,3/2} &= (0.19 \pm 0.03) \text{ GeV}^{5/2}, \end{aligned} \quad (12)$$

where the central value corresponds to  $T = 0.62$  GeV and  $\omega_c = 3.0$  GeV.

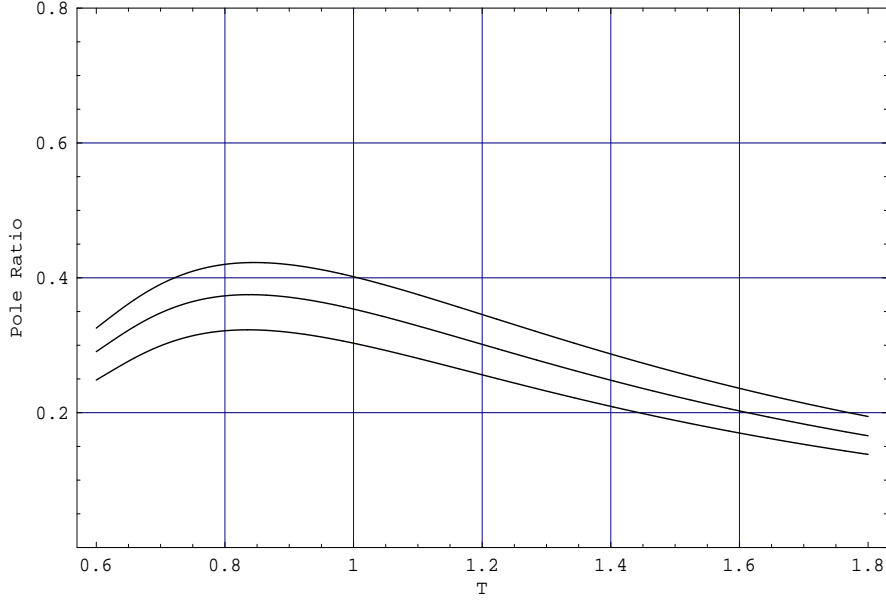


Figure 3: The variation of the ratio of the pole contribution and the perturbative piece of the  $(0^+, 1^+)$  doublet with  $T$  and  $\omega_c$  for the non-derivative currents. From top to bottom,  $\omega_c = 3.1, 2.9, 2.7$  GeV respectively.

### 2.3 $(0^+, 1^+)$ Doublet With Currents Without Derivative

Possible different currents for  $(0^+, 1^+)$  doublet are the non-derivative currents

$$\begin{aligned} J_{0,+,2}^\dagger &= \frac{1}{\sqrt{2}} \bar{h}_v s, \\ J_{1,+,2}^{\dagger\alpha} &= \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma_t^\alpha s. \end{aligned} \quad (13)$$

With the non-derivative currents the sum rule reads

$$\begin{aligned} f_{+,1/2}^2 e^{-2\bar{\Lambda}/T} &= \frac{3}{16\pi^2} \int_{2m_s}^{\omega_c} [\omega^2 - 2m_s\omega - 2m_s^2] e^{-\omega/T} d\omega \\ &+ \frac{1}{2} \langle \bar{s}s \rangle + \frac{m_s}{4T} \langle \bar{s}s \rangle - \frac{1}{8T^2} m_0^2 \langle \bar{s}s \rangle. \end{aligned} \quad (14)$$

Requiring that the condensate contribution is less than 30% of the perturbative term, we get  $T_{\min} = 0.75$  GeV. In Figure 3, we show the ratio of the pole term, i.e the sum of the cut-off perturbative term and condensation terms, for the sum rule (14). In the whole range of  $T > T_{\min}$ , this pole contribution is less than 40%. Hence, There is no stability window for this sum rule satisfying our criteria stated before.

If we arbitrarily loosen the analysis criteria and require the pole contribution is greater than 30% only, we get the working range  $0.75 < T < 1.2$  GeV. In Figure 4, the variation

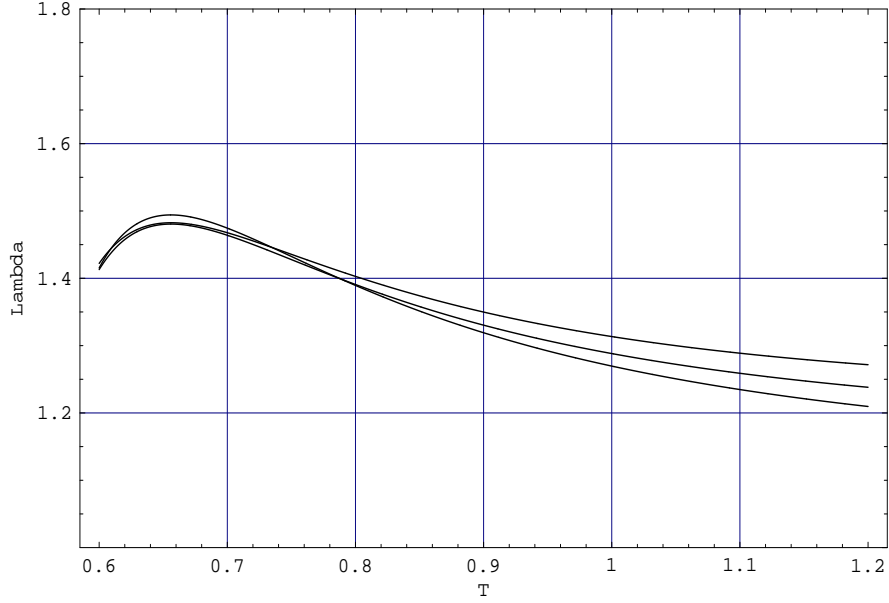


Figure 4: The variation of  $\bar{\Lambda}$  (in unit of GeV) of the  $(0^+, 1^+)$  doublet with  $T$  and  $\omega_c$  for the non-derivative currents. From top to bottom,  $\omega_c = 3.1, 2.9, 2.7$  GeV respectively.

of  $\bar{\Lambda}(1/2^+)$  with  $T$  and  $\omega_c$  is shown. Numerically,

$$\begin{aligned}\bar{\Lambda}(1/2^+) &= (1.30 \pm 0.15) \text{ GeV}, \\ f_{+,1/2} &= (0.39 \pm 0.05) \text{ GeV}^{3/2},\end{aligned}\tag{15}$$

where the central value corresponds to  $T = 1.0$  GeV and  $\omega_c = 2.9$  GeV. Because of the weaker criteria used these results are less reliable and will not be used in the final numerical results.

Here we would like to make some remarks. Usually, the currents with the least number of derivatives are used in QCD sum rule approaches. The sum rules then are less sensitive to the threshold energy  $\omega_c$ . However, it is pointed out in [17] that in the non-relativistic limit the coupling constant of these currents to the P wave states vanishes. If this coupling constant is suppressed due to this reason, the relative importance of the contribution of the  $DK$  and other states in continuum in the sum rules which are usually neglected would be enhanced. Besides, it is shown in [26, 27] by using soft pion theorem that the contribution of the  $D\pi$  continuum is large in the sum rule with the non-derivative current for the  $0^+$  state of the non-strange  $D$  system and significantly decreases the value of  $\bar{\Lambda}$ . Similar method of calculation is not good in the present case, but it indicates that the contribution of the  $DK$  continuum with the non-derivative current may be large too.

### 3 Sum rules at the $1/m_Q$ order

To the order of  $1/m_Q$ , the Lagrangian of HQET is

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S} + \mathcal{O}(1/m_Q^2), \quad (16)$$

where  $h_v(x)$  is the velocity-dependent effective field related to the original heavy-quark field  $Q(x)$  by

$$h_v(x) = e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x). \quad (17)$$

$\mathcal{K}$  is the kinetic operator defined as

$$\mathcal{K} = \bar{h}_v (iD_t)^2 h_v, \quad (18)$$

$\mathcal{S}$  is the chromo-magnetic operator

$$\mathcal{S} = \frac{g}{2} C_{\text{mag}}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (19)$$

where  $C_{\text{mag}} = \left( \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right)^{3/\beta_0}$ ,  $\beta_0 = 11 - 2n_f/3$ .

Considering  $\mathcal{O}(1/m_Q)$  corrections the pole term of the correlator on the hadron side becomes

$$\Pi(\omega)_{\text{pole}} = \frac{(f + \delta f)^2}{2(\bar{\Lambda} + \delta m) - \omega} = \frac{f^2}{2\bar{\Lambda} - \omega} - \frac{2\delta m f^2}{(2\bar{\Lambda} - \omega)^2} + \frac{2f\delta f}{2\bar{\Lambda} - \omega}, \quad (20)$$

where  $\delta m$  and  $\delta f$  are of order  $\mathcal{O}(1/m_Q)$ .

To extract  $\delta m$  in (20) we follow the approach of [23] to consider the three-point correlation functions

$$\delta_O \Pi_{j,P,i}^{\alpha_1 \dots \alpha_j, \beta_1 \dots \beta_j}(\omega, \omega') = i^2 \int d^4x d^4y e^{ik \cdot x - ik' \cdot y} \langle 0 | T \left( J_{j,P,i}^{\alpha_1 \dots \alpha_j}(x) O(0) J_{j,P,i}^{\dagger \beta_1 \dots \beta_j}(y) \right) | 0 \rangle, \quad (21)$$

where  $O = \mathcal{K}$  or  $\mathcal{S}$ . The scalar function corresponding to (21) can be represented as the double dispersion integral

$$\delta_O \Pi(\omega, \omega') = \frac{1}{\pi^2} \int \frac{\rho_O(s, s') ds ds'}{(s - \omega)(s' - \omega')}.$$

The pole parts of  $\delta_O \Pi(\omega, \omega')$ ,  $O = \mathcal{K}, \mathcal{S}$  are

$$\delta_{\mathcal{K}} \Pi(\omega, \omega')_{\text{pole}} = \frac{f^2 K}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \frac{f^2 G_{\mathcal{K}}(\omega')}{2\bar{\Lambda} - \omega} + \frac{f^2 G_{\mathcal{K}}(\omega)}{2\bar{\Lambda} - \omega'}, \quad (22)$$

$$\delta_{\mathcal{S}} \Pi(\omega, \omega')_{\text{pole}} = \frac{d_M f^2 \Sigma}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + d_M f^2 \left[ \frac{G_{\mathcal{S}}(\omega')}{2\bar{\Lambda} - \omega} + \frac{G_{\mathcal{S}}(\omega)}{2\bar{\Lambda} - \omega'} \right], \quad (23)$$



where

$$\begin{aligned} K_{j,P,j_l} &= \langle j, P, j_l | \bar{h}_v (iD_\perp)^2 h_v | j, P, j_l \rangle , \\ 2d_M \Sigma_{j,P,j_l} &= \langle j, P, j_l | \bar{h}_v g \sigma_{\mu\nu} G^{\mu\nu} h_v | j, P, j_l \rangle , \\ d_M &= d_{j,j_l}, \quad d_{j_l-1/2,j_l} = 2j_l + 2, \quad d_{j_l+1/2,j_l} = -2j_l. \end{aligned}$$

Letting  $\omega = \omega'$  in eqs. (22) and (23) and comparing with (20), one obtains [23]

$$\delta m = -\frac{1}{4m_Q}(K + d_M C_{mag} \Sigma) . \quad (24)$$

The single pole terms in (22) and (23) come from the region in which  $s(s') = 2\bar{\Lambda}$  and  $s'(s)$  is at the pole for a radial excited state or in the continuum. They are suppressed by making the double Borel transformation for both  $\omega$  and  $\omega'$ . The Borel parameters corresponding to  $\omega$  and  $\omega'$  are taken to be equal. One obtains thus the sum rules for  $K$  and  $\Sigma$  as

$$f^2 K e^{-2\bar{\Lambda}/T} = \int_{2m_s}^{\omega_c} \int_{2m_s}^{\omega_c} d\omega d\omega' e^{-(\omega+\omega')/2T} \rho_K(\omega, \omega') , \quad (25)$$

$$f^2 \Sigma e^{-2\bar{\Lambda}/T} = \int_{2m_s}^{\omega_c} \int_{2m_s}^{\omega_c} d\omega d\omega' e^{-(\omega+\omega')/2T} \rho_S(\omega, \omega') . \quad (26)$$

In this section we shall neglect the  $m_s$  corrections to  $K$  and  $\Sigma$ . We obtain for the  $j_l^P = \frac{1}{2}^+$  doublet with derivative currents

$$\begin{aligned} f_{+,1/2}'^2 K e^{-2\bar{\Lambda}/T} &= -\frac{3}{2^7 \pi^2} \int_{2m_s}^{\omega_c} \omega^6 e^{-\omega/T} d\omega + \frac{3}{2^4 \pi} \langle \alpha_s GG \rangle T^3 , \\ f_{+,1/2}'^2 \Sigma e^{-2\bar{\Lambda}/T} &= \int_{2m_s}^{\omega_c} \int_{2m_s}^{\omega_c} \rho(s, s') e^{-\frac{s+s'}{2T}} ds ds' + \frac{1}{48\pi} \langle \alpha_s GG \rangle T^3 , \end{aligned} \quad (27)$$

with

$$\rho(s, s') = \frac{1}{96} \frac{\alpha_s(2T)}{\pi^3} C_{mag} s s' \{ s'^2 (3s - s') \theta(s - s') + (s \leftrightarrow s') \} .$$

We can eliminate the dependence of the  $K$  and  $\Sigma$  on  $f_{+,1/2}$  and  $\bar{\Lambda}$  through dividing the above sum rules (27) by the sum rule (8). We use the same working windows as those of two-point sum rules for  $\bar{\Lambda}$  in evaluating  $\mathcal{O}(1/m_Q)$  corrections. Numerically we have

$$\begin{aligned} K_{1/2} &= (-1.60 \pm 0.30) \text{ GeV}^2 \\ \Sigma_{1/2} &= (0.28 \pm 0.05) \text{ GeV}^2 . \end{aligned} \quad (28)$$

The variations of  $K$  and  $\Sigma$  with  $T$  and  $\omega_c$  for the  $j_l^P = \frac{1}{2}^+$  doublet are shown in Figure 5 and 6, respectively.

For  $j_l^P = \frac{3}{2}^+$  doublet, we have

$$\begin{aligned} f_{+,3/2}^2 K e^{-2\bar{\Lambda}/T} &= -\frac{1}{2^7 \pi^2} \int_{2m_s}^{\omega_c} \omega^6 e^{-\omega/T} d\omega + \frac{7}{3 \times 2^5 \pi} \langle \alpha_s GG \rangle T^3 , \\ f_{+,3/2}^2 \Sigma e^{-2\bar{\Lambda}/T} &= \int_{2m_s}^{\omega_c} \int_{2m_s}^{\omega_c} \rho(s, s') e^{-\frac{s+s'}{2T}} ds ds' + \frac{1}{72\pi} \langle \alpha_s GG \rangle T^3 . \end{aligned} \quad (29)$$

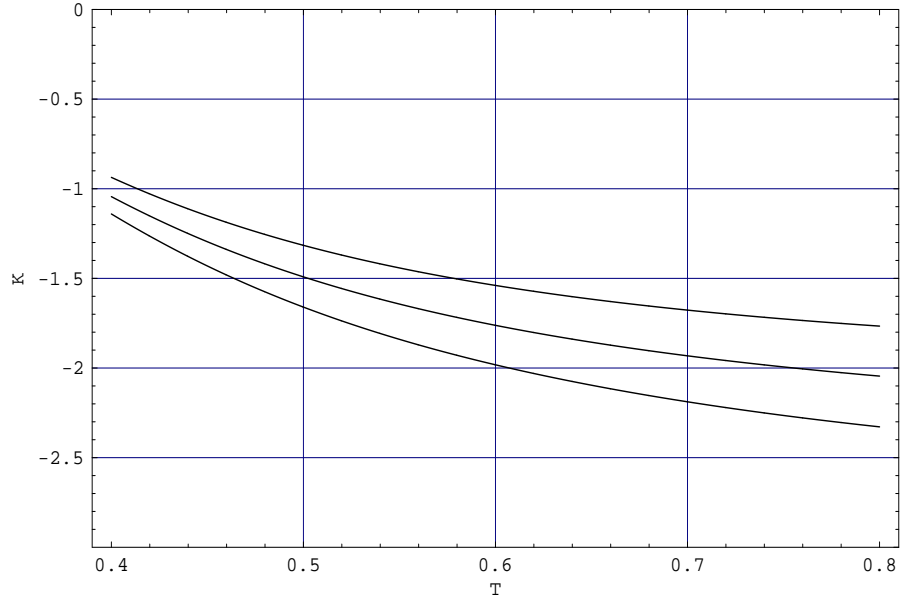


Figure 5: The variation of  $K_{1/2}$  (in unit of  $\text{GeV}^2$ ) of the  $(0^+, 1^+)$  doublet with  $T$  and  $\omega_c$  for the derivative currents. From top to bottom,  $\omega_c = 3.1, 2.9, 2.7$  GeV respectively.

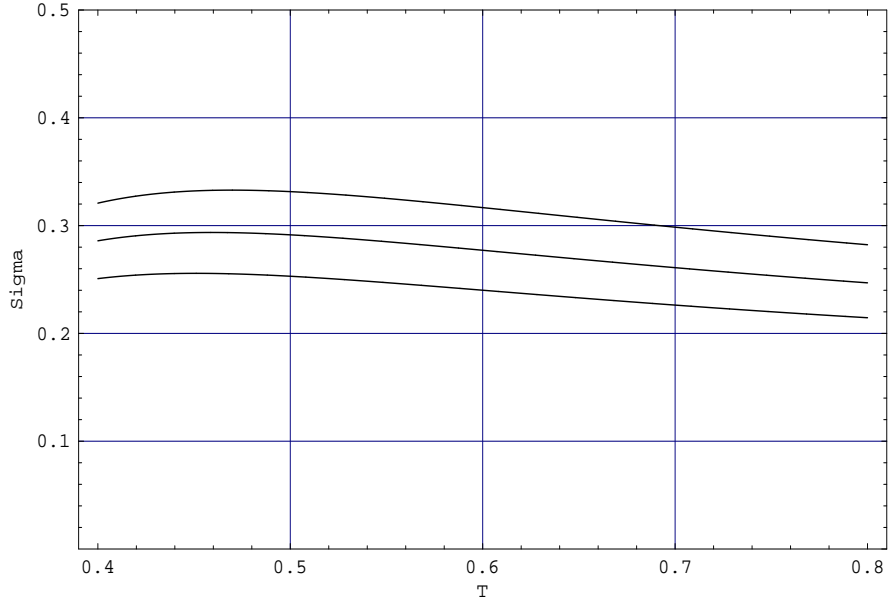


Figure 6: The variation of  $\Sigma_{1/2}$  (in unit of  $\text{GeV}^2$ ) of the  $(0^+, 1^+)$  doublet with  $T$  and  $\omega_c$  for the derivative currents. From top to bottom,  $\omega_c = 3.1, 2.9, 2.7$  GeV respectively.

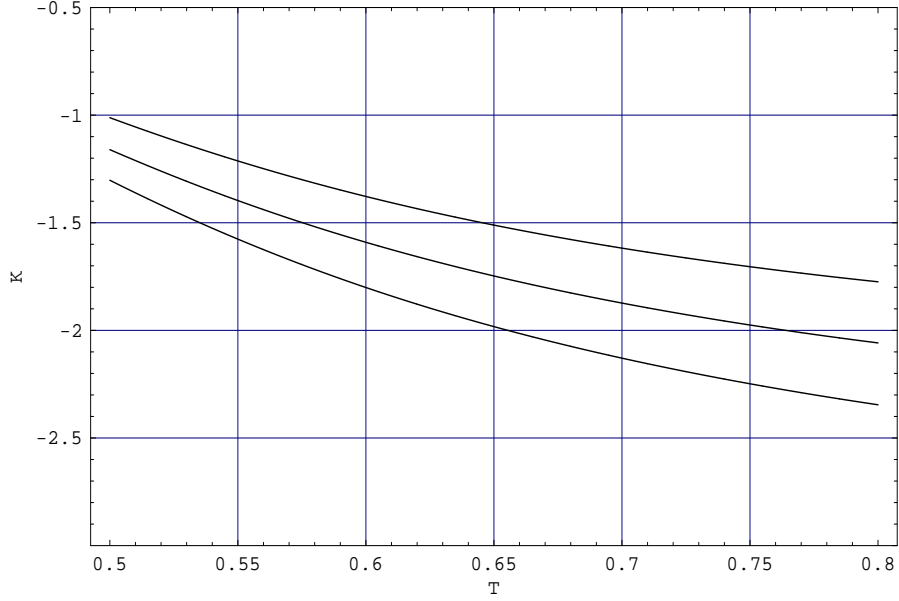


Figure 7: The variation of  $K_{3/2}$  (in unit of  $\text{GeV}^2$ ) of the  $(1^+, 2^+)$  doublet with  $T$  and  $\omega_c$ . From top to bottom,  $\omega_c = 3.2, 3.0, 2.8$  GeV respectively.

with

$$\rho(s, s') = \frac{1}{288} \frac{\alpha_s(2T)}{\pi^3} C_{mag} \left\{ s'^4 \left( s - \frac{3}{5} s' \right) \theta(s - s') + (s \leftrightarrow s') \right\}.$$

The variations of  $K$  and  $\Sigma$  with  $T$  and  $\omega_c$  for the  $j_l^P = \frac{3}{2}^+$  doublet are plotted in Figure 7 and 8 respectively. Numerically we have

$$\begin{aligned} K_{3/2} &= (-1.64 \pm 0.40) \text{ GeV}^2 \\ \Sigma_{3/2} &= (0.058 \pm 0.01) \text{ GeV}^2. \end{aligned} \quad (30)$$

The spin-symmetry violating term  $\mathcal{S}$  not only causes splitting of masses within the same doublet, but also causes mixing of states with the same  $j, P$  but different  $j_l$ . In Ref. [18] corrections from the mixing is found to be negligible. We omit this effect here.

## 4 Numerical Results and Discussions

We present our results for the  $\bar{c}s$  system assuming the HQET is good enough for excited  $D$  mesons. For the doublet  $(0^+, 1^+)$  with the derivative current, the weighted average mass is

$$\frac{1}{4} (m_{D_{s0}^*} + 3m_{D_{s1}^*}) = m_c + (0.86 \pm 0.10) + \frac{1}{m_c} [(0.40 \pm 0.08) \text{ GeV}^2].$$

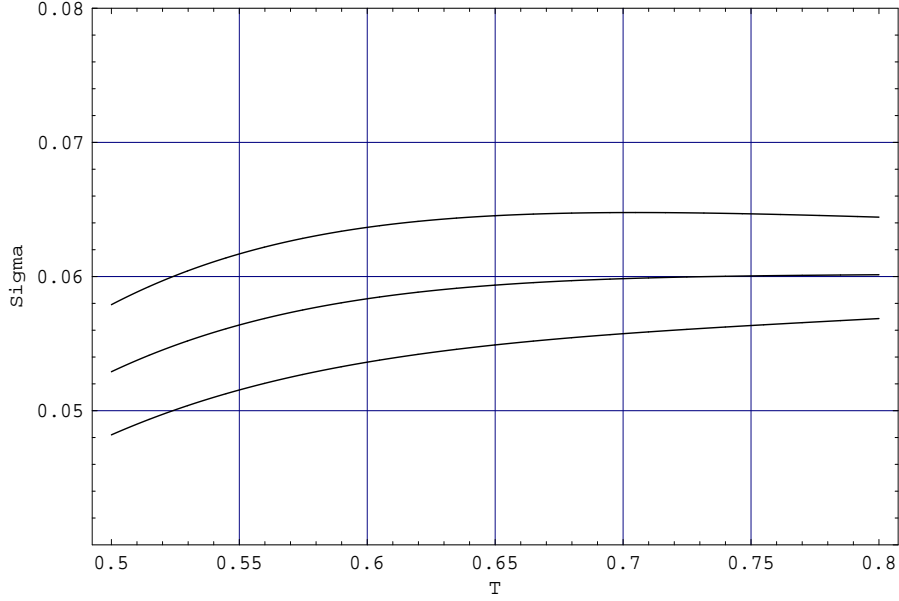


Figure 8: The variation of  $\Sigma_{3/2}$  (in unit of  $\text{GeV}^2$ ) of the  $(1^+, 2^+)$  doublet with  $T$  and  $\omega_c$ . From top to bottom,  $\omega_c = 3.2, 3.0, 2.8$  GeV respectively.

The mass splitting is

$$m_{D_{s1}^*} - m_{D_{s0}^*} = \frac{1}{m_c} [(0.28 \pm 0.05) \text{ GeV}^2] .$$

For the  $(1^+, 2^+)$  doublet we have

$$\frac{1}{8} (3m_{D_{s1}} + 5m_{D_{s2}^*}) = m_c + (0.83 \pm 0.10) + \frac{1}{m_c} [(0.41 \pm 0.10) \text{ GeV}^2] . \quad (31)$$

The  $1^+, 2^+$  mass splitting is

$$m_{D_{s2}^*} - m_{D_{s1}} = \frac{1}{m_c} [(0.116 \pm 0.06) \text{ GeV}^2] .$$

The results for  $\bar{b}s$  system are obtained by replacing  $m_c$  by  $m_b$  and multiplying  $\Sigma$  by 0.8 (since  $C_{mag} \approx 0.8$  for B system by using the values of  $\Lambda_{\text{QCD}}$  given in Sect. 2) in above equations.

Choosing  $m_c$  to fit the experimental value

$$\frac{1}{8} (3m_{D_{s1}} + 5m_{D_{s2}^*}) = 2.56 \text{ GeV},$$

where we again neglect the mixing between two  $1^+$  states, we obtain  $m_c = 1.44$  GeV. Using this  $m_c$  value we obtain the following numerical results. The  $1^+, 2^+$  mass splitting is

$$m_{D_{s2}^*} - m_{D_{s1}} = (0.080 \pm 0.042) \text{ GeV}$$

which is consistent with the experimental value 37 MeV within the large theoretical uncertainty. Experimentally, the mass splitting in the  $(1^+, 2^+)$  doublet in  $D_s$  system is almost equal to that in  $D$  system. This justifies neglecting the  $m_s$  correction to the  $\Sigma$  term in our calculation. For  $(0^+, 1^+)$  doublet, the weighted average mass is

$$\frac{1}{4} (m_{D_{s0}^*} + 3m_{D_{s1}^*}) = (2.57 \pm 0.12) \text{ GeV}.$$

The mass splitting is

$$m_{D_{s1}^*} - m_{D_{s0}^*} = (0.19 \pm 0.04) \text{ GeV}. \quad (32)$$

Therefore, the  $0^+$  mass is predicted to be  $m_{0^+} = (2.42 \pm 0.13) \text{ GeV}$ . This is consistent with the experimental value 2.317 GeV, though the central value is 100 MeV larger than data. If the  $1^+$   $D_s$  state of mass 2.460 GeV found by CLEO [2, 1] is assigned as the other member of the  $(0^+, 1^+)$  doublet, then the observed mass splitting 0.143 GeV is consistent with (32). Notice that the  $\Sigma_{1/2}$  value in Fig. 6 changes very slowly between  $T = 0.35 - 0.8$  GeV. Therefore the result (32) is insensitive to the stability window used.

We would like to note that the stability of the sum rules (in particular, those for the K and  $\Sigma$ ) obtained by us is not as good as those for the ground states, as can be seen from the figures. And for the charm flavor hadrons, the  $1/m_c$  corrections are significant. So the predictions on masses have a relatively large uncertainties, which are estimated by given errors.

If we replace the strange quark condensate by up/down quark condensate and  $m_s$  by the zero up/down quark mass, we can extract the non-strange excited  $D$  meson masses. For  $(1^+, 2^+)$  doublet, the sum rule window is  $0.57 < T < 0.67 \text{ GeV}$  for  $\omega_c = 2.8 - 3.2 \text{ GeV}$ . The results are

$$\begin{aligned} \Lambda_{3/2} &= (0.73 \pm 0.08) \text{ GeV}, \\ K_{3/2} &= -(1.6 \pm 0.4) \text{ GeV}^2, \\ \Sigma_{3/2} &= (0.057 \pm 0.01) \text{ GeV}^2 \end{aligned} \quad (33)$$

with the central value at  $T = 0.62 \text{ GeV}$ . For  $(0^+, 1^+)$  doublet, the sum rule window is  $0.46 < T < 0.6 \text{ GeV}$  for  $\omega_c = 2.7 - 3.1 \text{ GeV}$ . The results are

$$\begin{aligned} \Lambda_{1/2} &= (0.79 \pm 0.08) \text{ GeV}, \\ K_{1/2} &= -(1.57 \pm 0.4) \text{ GeV}^2, \\ \Sigma_{1/2} &= (0.286 \pm 0.05) \text{ GeV}^2 \end{aligned} \quad (34)$$

with the central value at  $T = 0.53 \text{ GeV}$ . The effect of strange quark is to make the mass of  $D_s$  mesons a little bit larger, as expected. Experimentally, Belle collaboration found  $m_{D_0} = (2290 \pm 22 \pm 20) \text{ MeV}$  and  $m_{D_1^*} = (2400 \pm 30 \pm 20) \text{ MeV}$  recently [29] while CLEO collaboration found  $m_{D_1^*} = 2400^{+48}_{-42} \text{ MeV}$  [30]. The  $j_{3/2}$  non-strange doublet mass is known precisely [28],  $m_{D_2} = 2460 \text{ MeV}$  and  $m_{D_1} = 2420 \text{ MeV}$ . Our results (33)-(34) are consistent with the experimental data within theoretical uncertainty.

In summary, we have calculated the masses of the excited  $(0^+, 1^+)$  and  $(1^+, 2^+)$  doublets for the  $\bar{c}s$  system to the  $1/m_Q$  order in the HQET sum rules. The numerical results

imply that the  $D_{sJ}(2317)$  and  $D_{sJ}(2460)$  observed by BABAR and CLEO can be consistently identified as the  $(0^+, 1^+)$  doublet with  $j_l = \frac{1}{2}^+$  within the theoretical uncertainty. Especially, the mass splitting in the  $(0^+, 1^+)$  doublet is reproduced quite well. The repulsion between these states and the  $DK$  and  $DK^*$  continuum may help to lower their masses. In the framework of the sum rule this effect should come from the contribution from the  $DK$  and  $DK^*$  continuum to the dispersion integral.

*Acknowledgments:* This project was supported by the National Natural Science Foundation of China, BEPC Opening Project, Ministry of Education of China, FANEDD and SRF for ROCS, SEM.

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